My research focuses on the computable and proof-theoretic content of mathematical theorems, primarily from Ramsey theory. Once a theorem is proven using strong axioms, natural questions arise: Were the axioms used necessary? How complex are the objects whose existence it entails? Which statements are provable when it is itself adopted as an axiom?

Reverse mathematics is a program devoted to answering such questions. Working modulo the weak base theory RCA_0 of second-order arithmetic, which restricts set existence to computable sets and allows only limited induction, we aim to compare the strength of second-order statements.

Computable analysis and algorithmic randomness

The truth of a second-order statement may imply the existence of non-computable sets, making it natural to compare theorem strength by the sets whose existence they entail. The (hyper)arithmetical hierarchy, whose levels correspond to iterates of the halting problem, provides one such measure.

The hierarchy of Ramsey theorems for colorings of n-tuples collapses at level 3; above this level, every statement implies the existence of all arithmetical sets and is thus equivalent to the theory ACA_0 modulo RCA_0. This is not the case for several weakenings of the Ramsey theorem (the Free Set, Thin Set and Rainbow Ramsey theorems), as shown by Wang [15], since they admit the so-called *(strong) cone avoidance* property. We showed that this behavior persists for a generalization of the Rainbow Ramsey theorem to tuples of unbounded size, but fails dramatically for the other principles, whose generalizations imply the existence of the ω-jump $\emptyset^{(\omega)}$.

Theorem 1 ([2])**.** $\mathsf{RRT}^{!\omega}$ *admits strong cone avoidance.*

Other hierarchies arise from the study of algorithmic randomness, where sets are compared according to the extent to which they satisfy properties that hold almost everywhere. For a problem, the existence of probabilistic solutions can be regarded as an indicator of its computability-theoretic weakness.

When studying weakenings of the Ramsey theorem for pairs and two colors (RT^2_2) where we try to avoid a given pattern p in the resulting infinite clique (rather than searching for an infinite monochromatic clique), we identified a distinguished subclass of permutations, the separable permutations, as being exactly those patterns whose avoidance is the hardest (in combination with the results of Mimouni and Levy Patey [14]). More precisely, we showed that their corresponding dual statement (namely, starting from a coloring already avoiding a pattern and finding an infinite monochromatic clique) admits probabilistic solutions, measured here by the ability to compute diagonally non-computable functions (DNC).

Theorem 2 ([6])**.** *Let* $\mathbf{d}$ *be of* $\emptyset'$-DNC *degree. Then* $\mathbf{d}$ *computes a solution to computable instances of* $\mathsf{coRT}^2_2(p)$ *if and only if* p *is a separable permutation.*

Conservation and first-order part

A statement P is said to be conservative over a theory T for a class Γ of formulas if for every proof in $T + \mathsf{P}$ of some $\varphi \in \Gamma$, there exists a proof of φ in T. One major open question in the reverse mathematical analysis of theorems from Ramsey theory is the identification of the *first-order* part of Ramsey theorem for pairs and two colors (RT^2_2), that is, characterizing the set of first-order statements provable in $\mathsf{RCA}_0 + \mathsf{RT}^2_2$. It is known (Hirst [5]) that this first-order part contains induction for Δ^0_2-formulas ($\mathsf{I}\Delta^0_2$), and is strictly included in the consequences of Σ^0_2 induction $\mathsf{I}\Sigma^0_2$ (Chong et al. [3]). The question of whether it is exactly $\mathsf{I}\Delta^0_2$ is still open.

Together with Ludovic Patey and Keita Yokoyama, we made several contributions on the subject.

Theorem 3 ([10]). RT^2_2 *is* Π^1_1*-conservative over* $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2 + \mathsf{WF}(\epsilon_0)$.

As $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2 + \mathsf{WF}(\epsilon_0) \not\vdash \mathsf{I}\Sigma^0_2$, this is an improvement of Chong et al. [3]. The proof relies on the method of ω-extension, whereby a (possibly non-standard) model of $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2 + \mathsf{WF}(\epsilon_0)$ is extended to a model of RT^2_2 through the addition of new sets. This yields a Σ^1_1-elementary extension of the initial model, preserving the validity of first-order sentences between the two models.

We also considered the construction of new models using the techniques of cuts to change their first-order part. Using new combinatorial techniques based on the notion of α-largeness of Ketonen and Solovay [8], we extended the $\forall\Pi^0_3$ conservation result of Patey and Yokoyama [13] to a $\forall\Pi^0_4$ conservation (where a $\forall\Gamma$ formula is a formula of the form $\forall X \varphi(X)$ for $\varphi \in \Gamma$):

Theorem 4 ([12]). RT^2_2 *is* $\forall\Pi^0_4$*-conservative over* $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2$.

This result is significant in that a $\forall\Pi^0_5$ conservation over $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2$ would yield full arithmetical conservation, as shown by Fiori-Carones et al. [4].

The proof techniques of this article were later used in the study of tree partition theorems, with the following conservation result for the ordered variable word theorem OVW.

Theorem 5 ([11]). OVW *is* $\forall\Pi^0_4$*-conservative over* $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2$.

In particular, OVW does not imply ACA_0 over RCA_0, making it the first principle for which the only known separation from ACA_0 involves non-standard models.

Proof size

Once conservation of a statement P over a theory T has been established, one may ask whether the stronger theory $\mathsf{P} + T$ allows for shorter proofs than T alone.

By adapting the general framework of forcing interpretations introduced by Avigad [1] and used by Kołodziejczyk et al. [9] on the $\forall\Pi^0_3$ conservation of RT^2_2 over RCA_0, we proved that the conservation result stated in Theorem 4 can be effectivized with at most a polynomial increase in proof size. The proof relies on heavy combinatorial arguments to derive appropriate bounds for variants of the finite Ramsey theorem.

Theorem 6 ([7]). $\mathsf{RCA}_0 + \mathsf{I}\Delta^0_2$ *polynomially simulates* $\mathsf{RCA}_0 + \mathsf{RT}^2_2$ *with respect to* $\forall\Pi^0_4$ *sentences.*

Interest

Apart from pursuing my investigations in the aforementioned areas, I am also interested in broadening my knowledge of related fields of study.

My work in [10], [12] and [7] has led me to develop a particular interest in ordinal analysis, especially in the context of weak arithmetic theories where standard characterizations of proof-theoretic ordinals may fail to apply.

I am also interested in the consequences of the isomorphism theorem of Fiori-Carones et al. [4], which suggests that, in certain respects, the model theory of structures in which induction fails may be better behaved than expected.

I plan to study statements of peculiar proof-theoretic strength. So far, my work has primarily focused on combinatorial principles with weak reverse mathematical strength (mostly below ACA_0) arising from Ramsey theory, but I am also interested in going higher, notably with the study of the theory of well- and better-quasi-orders, as well as certain set-theoretic statements.

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